MATHEMATICS OF COMPUTATION, VOLUME 27, NUMBER 122, APRIL, 1973

On the Gaussian Primes on the Line Im$(X) = 1$

By M. C. Wunderlich*

Abstract. This paper contains a summary table of the author's computation of the Gaussian primes of the form $a + i$. For the values $x = 1000$, 10000, 100000, 180000, and 500000 (500000) 14000000, the following values are tabulated: $G(x)$, the numbers of Gaussisn primes $a + i$ with $a \leqq 14000000$; $\pi_1(x)$, the number of primes $\leqq x$ congruent to 1 mod 4; $\pi_3(x)$, the number of primes $\leqq x$ congruent to 3 mod 4; and $G(x)/\pi_3(x)$.

One of the well-known 'hard' problems in number theory which has not yet been solved is to determine whether there are infinitely many Gaussian primes of the form $a + i$. Since a Gaussian integer $a + bi$ for $b \neq 0$ is a Gaussian prime iff $a^2 + b^2$ is 2 or a rational prime congruent to 1 modulo 4, the problem is equivalent to determining whether there are infinitely many rational primes of the form $a^2 + 1$. In 1960, Daniel Shanks [2] employed a p-adic sieving procedure to completely factor all numbers of the form $a^2 + 1$ for $a \leqq 180,000$. The purpose of this note is to extend the results of Shanks' computations to $a \leqq 14,000,000$.

Table I summarizes the results of our computations. $G(x)$ is the number of Gaussian primes of the form $a + i$ for $0 < a \leqq x$. For comparative purposes, we list $\pi_1(x)$

TABLE I

x	$G(x)$	$\pi_1(x)$	$\pi_3(x)$	$G(x)/\pi_3(x)$
1 000	112	80	87	1.28736
10 000	841	609	619	1.35864
100 000	6 656	4 783	4 808	1.38436
180 000	11 223	8 163	8 178	1.37234
500 000	28 563	20 731	20 806	1.37283
1 000 000	54 110	39 175	39 322	1.37607
1 500 000	78 515	57 022	57 092	1.37524
2 000 000	102 205	74 416	74 516	1.37159
2 500 000	125 481	91 432	91 639	1.36930
3 000 000	148 655	108 283	108 532	1.36969
3 500 000	171 556	124 996	125 153	1.37077
4 000 000	194 230	141 502	141 643	1.37126
4 500 000	216 808	157 924	158 023	1.37200
5 000 000	239 185	174 193	174 319	1.37207
5 500 000	261 392	190 282	190 517	1.37201

Received November 3, 1971, revised March 13, 1972.
AMS (MOS) subject classifications (1970). Primary 10-04, 10H20.
* The research for this paper was partially supported by NSF grant GP-23299.

TABLE I (*continued*)

x	$G(x)$	$\pi_1(x)$	$\pi_3(x)$	$G(x)/\pi_3(x)$
6 000 000	283 312	206 332	206 516	1.37187
6 500 000	305 204	222 272	222 484	1.37180
7 000 000	327 066	238 260	238 387	1.37203
7 500 000	348 764	253 973	254 287	1.37154
8 000 000	370 461	269 759	270 017	1.37199
8 500 000	392 138	285 395	285 723	1.37244
9 000 000	413 756	301 100	301 388	1.37284
9 500 000	435 105	316 641	316 936	1.37285
10 000 000	456 362	332 180	332 398	1.37294
10 500 000	477 708	347 664	347 944	1.37295
11 000 000	498 760	363 079	363 437	1.37234
11 500 000	519 845	378 517	378 770	1.37246
12 000 000	540 867	393 992	394 067	1.37253
12 500 000	561 869	409 308	409 394	1.37244
13 000 000	582 580	424 568	424 683	1.37180
13 500 000	603 626	439 790	439 849	1.37235
14 000 000	624 535	454 973	455 103	1.37229

and $\pi_3(x)$ which are the number of rational primes $p \leqq x$ congruent to 1 modulo 4 and 3 modulo 4, respectively. $\pi_1(x)$ is a measure of the amount of work done to perform the calculation up to x, since a sieve was used which was executed once for every $p = 1 \pmod 4$. Hardy and Littlewood [1] conjectured that $G(x)/\pi_3(x)$ should converge to 1.3728 as $x \to \infty$ and the last column in the table shows that this figure is being approximated quite well up to 14,000,000. The second, third, and fourth rows of the table contain results which agree with Shanks' published figures.

Department of Mathematics
Northern Illinois University
De Kalb, Illinois 60625

1. G. H. HARDY & J. E. LITTLEWOOD, "Partitio numerorum III: On the expression of a number as a sum of primes," *Acta. Math.*, v. 44, 1923, p. 48.
2. DANIEL SHANKS, "A sieve method for factoring numbers of the form $n^2 + 1$," *MTAC*, v. 13, 1959, pp. 78–86. MR **21** #4520.