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# Factorizations of Cunningham numbers with bases 13 to 99: millennium edition

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# Factorizations of Cunningham Numbers with Bases $13$ to $99$: Millennium Edition*

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ABSTRACT

This Report updates the tables of factorizations of $a^n \pm 1$ for $13 \le a < 100$, previously published as CWI Report NM-R9212 (June 1992) and updated in CWI Report NM-R9419 (Update 1, September 1994) and CWI Report NM-9609 (Update 2, March 1996). A total of 951 new entries in the tables are given here. The factorizations are now complete for $n < 76$, and there are no composite cofactors smaller than $10^{102}$.

This "Millennium edition" gives internet pointers to electronic versions of the complete tables incorporating all updates. A file containing only the new updates, a file containing factorizations for an extended table range, and a file of factors, are also available on the internet.

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*Note:* This report has also appeared as Technical Report TR-14-00 of the Computing Laboratory of Oxford University, December 2000 (see http://web.comlab.ox.ac.uk/oucl/publications/tr/index.html). The research of Cavallar, Montgomery, and Te Riele was carried out under project MAS2.2 "Computational number theory and data security".

* Incorporating *Factorizations of* $a^n \pm 1$, $13 \le a < 100$*: Update 3*

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## 1. Introduction

For many years there has been an interest in the prime factors of numbers of the form $a^n \pm 1$, where $a$ is a small integer (the *base*) and $n$ is a positive exponent. Such numbers often arise. For example, if $a$ is prime then there is a finite field $F$ with $a^n$ elements, and the multiplicative group of $F$ has $a^n - 1$ elements. Also, for prime $a$ the sum of divisors of $a^n$ is $\sigma(a^n) = (a^{n+1} - 1)/(a - 1)$. Numbers of the form $a^n + 1$ arise as factors of $a^{2n} - 1$ and in other ways.

An extensive table of factors of $a^n \pm 1$ for $a \le 12$ has been published by Brillhart *et al* [11]. The computation of these tables is referred to as the *Cunningham Project* in recognition of the pioneering computations of Cunningham and Woodall [12]. For a history, see the Introduction in [11].

The tables [11] are limited to $a \le 12$, but many applications require larger bases. In June 1992 tables covering the range $13 \le a < 100$ were published [8]. The exponents $n$ satisfied $a^n < 10^{255}$ if $a < 30$, and $n \le 100$ if $a \ge 30$. An update [9] containing 780 new factorizations (with the same limits for $a$ and $n$) was published in September 1994, and a second update [10] containing 760 new factorizations was published in March 1996. These factorizations are now incorporated in the *Magma* package [3].

Since the second update [10], many new factors have been found. The factorizations are now complete for $n \le 75$, and there are no composite cofactors with fewer than 103 digits[1]. This report includes all the new (complete or partial) factorizations found from the publication of [10] to 31 December 2000. Altogether, 951 new (complete or partial) factorizations are listed, involving 1098 new factors[2]. Table 1 summarizes progress since the publication of the original tables [8]. "Update 3" refers to this Report.

Table 1: Statistics regarding the Tables and Updates

| Tables | Date | Smallest composite | Complete to exponent | Total entries |
|---|---|---|---|---|
| Original | June 1992 | 81 digits | 46 | 13882 |
| Update 1 | Sept. 1994 | 87 digits | 58 | 780 |
| Update 2 | March 1996 | 95 digits | 66 | 760 |
| Update 3 | Dec. 2000 | 103 digits | 75 | 951 |

Table 2 shows the number of prime factors of different sizes found for Updates 1–3 (excluding large factors obtained by division). The median sizes are 26 digits for Update 1, 29–30 digits for Update 2, and 33 digits for Update 3. The *smallest* new factor is 20 digits for Update 3 (compare 14 digits for Update 2). We would be surprised if many factors of less than 25 digits are still to be found. The *largest* new penultimate factor is 60 digits for Update 3 (compare 56 digits for Update 2).

## 2. Format of the Tables

The format of the Tables is the same as in [8]. For each base $a$, not a perfect power, in the range $13 \le a < 100$, we give two separate tables –

[1] "digits" always means "decimal digits".

[2] Here and elsewhere we do not count large factors which are obtained by division by other factors.

Table 2: Distribution of Factors

| Digits | Update 1 | Update 2 | Update 3 |
|---|---|---|---|
| 10–14 | 0 | 1 | 0 |
| 15–19 | 17 | 23 | 0 |
| 20–24 | 333 | 144 | 24 |
| 25–29 | 329 | 273 | 242 |
| 30–34 | 154 | 197 | 322 |
| 35–39 | 72 | 99 | 181 |
| 40–44 | 44 | 89 | 134 |
| 45–49 | 9 | 39 | 107 |
| 50–54 | 0 | 14 | 63 |
| 55–59 | 1 | 3 | 15 |
| 60–64 | 0 | 0 | 10 |
| Total | 959 | 882 | 1098 |

*Table* $a-$: factorizations of $a^n - 1$, $n$ odd.

*Table* $a+$: factorizations of $a^n + 1$.

The exponent ranges are as in [8] –

$13 \le a < 30$, exponents $n$ such that $a^n < 10^{255}$.

$30 \le a < 100$, exponents $n \le 100$.

The entries are similar in format to those of the "short" tables in [11]. All known factors, including algebraic and Aurifeuillian [4] factors, are listed. Factors which are given as decimal numbers are primes. Exponents are indicated by a hat (^), for example "2^3" means $2^3$. Multiplication is indicated by a period (.), for example $3^3 + 1 = 2^2 7$ is written as "2^2.7". A period at the end of a line implies that the factorization is continued on the next line. An underscore (_) at the end of a line means that a (large) factor is continued on the next line (see, for example, the entry for $19^{177} - 1$).

The largest factor of $a^n \pm 1$ may be found by division by the smaller factors. Thus, such factors are abbreviated. The notation $p_{xy}$ or "`pxy`" means a prime factor of `xy` digits. For example, the prime 1238926361552897 might be abbreviated as `p16`. Similarly, the notation $c_{xy}$ or "`cxy`" means a composite number of `xy` digits.

## 3. Availability of Tables, Updates and Factors

The changes since Update 2 are available by anonymous ftp from `ftp://ftp.comlab.ox.ac.uk/pub/Documents/techpapers/Richard.Brent/rpb134u3.txt.gz` (a compressed text file). This file includes comments on the person and method responsible for finding each factor (if there is no attribution, the factor was found by one of the authors).

For technical reasons, in this CWI Report we only give the complete Tables 13−, 13+, 99−, and 99+ for the original table range. The complete tables for $13 \le a < 100$ incorporating Updates 1–3, and a list of factors, are available online: see `http://www.comlab.ox.ac.uk/oucl/work/richard.brent/factors.html`. See also: `http://www.cwi.nl/ftp/herman/`

`Cunn2tot.txt.Z`. The restriction $n \leq 100$ for bases $a \geq 30$ has been relaxed in the Oxford tables; there it is only required that $a^n < 10^{255}$. For this extended table range, the smallest composite has 102 digits.

## 4. Factorization Methods

Since Update 2 we have attempted to factor the remaining composite numbers in the tables by using the *elliptic curve method* (ECM). Sometimes ECM is successful in finding one or more factors. If the factorization can not be completed by ECM, but the remaining composite part is sufficiently small, we use the *multiple polynomial quadratic sieve* (MPQS) method to complete the factorization. In some cases we prefer to use the *number field sieve* (NFS) if it is predicted to be faster than MPQS[3].

We do not describe ECM, MPQS or NFS here. The reader should refer to [16, 17, 19] for a general description of ECM, to [2, 23] for MPQS, and to [15, 13, 21] for NFS. A recent survey is [7]. The particular implementations of ECM by Brent and Montgomery are described in [6, 18].

Table 3 shows the number of factors found by several methods in the preparation of Updates 1–3. For ECM and MPQS these only include penultimate factors of at least 30 digits. An increase in the use of SNFS and decline in the use of Pollard's $p \pm 1$ methods [22] is evident. There is also a marked increase in the number of large (at least 30-digit) factors found by ECM. Most of the new factors found by MPQS and SNFS are large because these methods are only used after ECM has been tried. In fact, since Update 2, MPQS and SNFS did not find any factor with less than thirty digits, because such factors had already been found by ECM. The largest factor found by ECM was a 52-digit factor of $96^{98} + 1$ (see [5]).

Table 3: Factors Found by Different Methods

| | Pollard $p-1$ | Pollard $p+1$ | ECM (30D+) | MPQS (30D+) | NFS |
|---|---|---|---|---|---|
| Update 1 | 38 | 16 | 69 | 157 | 37 |
| Update 2 | 0 | 3 | 151 | 155 | 136 |
| Update 3 | 0 | 3 | 423 | 129 | 279 |

## 5. First Holes

A "first hole" is the first composite number occurring in a table. Thus, each table of factorizations is complete up to, but not including, its first hole. Table 4 lists the exponents of the current first holes for $2 \leq a < 100$ (the range $2 \leq a \leq 12$ is included for the sake of comparison). For example, the first holes in the tables for $a = 17$ occur for exponents 137 and 118. In fact, first holes such as $17^{118} + 1 = 2 \cdot 5 \cdot 29 \cdot 7789 \cdot c_{139}$ are good candidates for factorization by SNFS.

[3] The choice depends upon the size of the known non-algebraic factors of the number $a^n \pm 1$. We normally use the *special* number field sieve (SNFS), but in at least one case ($17^{186} + 1$) the *general* number field sieve (GNFS) was used (a contribution by Couveignes, Granboulan, Hoogvorst and Nguyen).

Table 4: Exponents of First Holes for $2 \le a \le 99$

| $a$ | $-$ | $+$ | $a$ | $-$ | $+$ | $a$ | $-$ | $+$ | $a$ | $-$ | $+$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 641 | 617 | 3 | 379 | 382 | 5 | 307 | 283 | 6 | 251 | 232 |
| 7 | 227 | 214 | 10 | 197 | 223 | 11 | 191 | 181 | 12 | 173 | 172 |
| 13 | 161 | 151 | 14 | 149 | 134 | 15 | 127 | 122 | 17 | 137 | 118 |
| 18 | 131 | 121 | 19 | 155 | 113 | 20 | 149 | 106 | 21 | 125 | 128 |
| 22 | 103 | 116 | 23 | 101 | 101 | 24 | 101 | 107 | 26 | 107 | 103 |
| 28 | 103 | 106 | 29 | 101 | 112 | 30 | 103 | 103 | 31 | 97 | 113 |
| 33 | 103 | 89 | 34 | 103 | 101 | 35 | 97 | 103 | 37 | 89 | 97 |
| 38 | 101 | 86 | 39 | 89 | 89 | 40 | 97 | 97 | 41 | 101 | 89 |
| 42 | 115 | 86 | 43 | 101 | 89 | 44 | 103 | 94 | 45 | 83 | 92 |
| 46 | 101 | 82 | 47 | 89 | 86 | 48 | 107 | 94 | 50 | 89 | 97 |
| 51 | 97 | 83 | 52 | 83 | 82 | 53 | 89 | 88 | 54 | 107 | 79 |
| 55 | 107 | 86 | 56 | 83 | 79 | 57 | 83 | 79 | 58 | 83 | 76 |
| 59 | 97 | 79 | 60 | 79 | 86 | 61 | 79 | 94 | 62 | 79 | 82 |
| 63 | 83 | 83 | 65 | 79 | 79 | 66 | 97 | 86 | 67 | 83 | 82 |
| 68 | 79 | 76 | 69 | 83 | 83 | 70 | 83 | 89 | 71 | 89 | 76 |
| 72 | 79 | 83 | 73 | 79 | 83 | 74 | 89 | 82 | 75 | 79 | 79 |
| 76 | 103 | 79 | 77 | 89 | 86 | 78 | 79 | 79 | 79 | 97 | 83 |
| 80 | 83 | 82 | 82 | 83 | 79 | 83 | 79 | 82 | 84 | 79 | 76 |
| 85 | 83 | 76 | 86 | 79 | 79 | 87 | 97 | 83 | 88 | 79 | 92 |
| 89 | 83 | 83 | 90 | 83 | 79 | 91 | 83 | 92 | 92 | 83 | 82 |
| 93 | 79 | 82 | 94 | 79 | 76 | 95 | 79 | 79 | 96 | 83 | 79 |
| 97 | 83 | 82 | 98 | 95 | 79 | 99 | 89 | 88 | | | |

## 6. Probable Primes

Numbers listed as prime have not in all cases been rigorously proved to be prime; they may merely have passed a probabilistic primality test [14]. There is a positive but extremely small probability that a composite number will pass such a test and be mistaken for a prime. In applications where it is essential for primality to be proven rigorously, one should apply an algorithm such as Morain's elliptic curve primality test [1, 20], which can easily prove or disprove the primality of numbers of the size considered here.

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## The Tables

For technical reasons, in this CWI Report we only give the example Tables 13−, 13+, 99−, and 99+. For pointers to online versions of the Tables $13-, 13+, 14-, \ldots, 98+, 99-, 99+$, see §3.

Table 13-

| | | |
|---|---|---|
| 13 | 1- | 2^2.3 |
| 13 | 3- | 2^2.3^2.61 |
| 13 | 5- | 2^2.3.p5 |
| 13 | 7- | 2^2.3.p7 |
| 13 | 9- | 2^2.3^3.61.p7 |
| 13 | 11- | 2^2.3.23.419.859.p5 |
| 13 | 13- | 2^2.3.53.264031.p7 |
| 13 | 15- | 2^2.3^2.61.4651.30941.p6 |
| 13 | 17- | 2^2.3.103.443.p14 |
| 13 | 19- | 2^2.3.12865927.p13 |
| 13 | 21- | 2^2.3^2.43.61.337.547.2714377.p7 |
| 13 | 23- | 2^2.3.1381.p22 |
| 13 | 25- | 2^2.3.701.9851.30941.p16 |
| 13 | 27- | 2^2.3^4.61.650971.1609669.p14 |
| 13 | 29- | 2^2.3.1973.2843.3539.p21 |
| 13 | 31- | 2^2.3.311.1117.p28 |
| 13 | 33- | 2^2.3^2.23.61.419.859.18041.p23 |
| 13 | 35- | 2^2.3.211.30941.5229043.3357897971.p15 |
| 13 | 37- | 2^2.3.1481.67495678093.4287755796749.p14 |
| 13 | 39- | 2^2.3^2.53.61.79.1093.4603.21841.264031.1803647.p14 |
| 13 | 41- | 2^2.3.6740847065723.p32 |
| 13 | 43- | 2^2.3.119627.p42 |
| 13 | 45- | 2^2.3^3.61.181.4651.30941.161971.1609669.p25 |
| 13 | 47- | 2^2.3.183959.19216136497.p36 |
| 13 | 49- | 2^2.3.1667.5229043.28082195177.p34 |
| 13 | 51- | 2^2.3^2.61.103.443.763879.15798461357509.p30 |
| 13 | 53- | 2^2.3.107.194723.18954118094396 9.8403659652641423.p21 |
| 13 | 55- | 2^2.3.23.419.859.2861.18041.30941.13545148572117361.p25 |
| 13 | 57- | 2^2.3^2.61.12865927.796956375829.9468940004449.p29 |
| 13 | 59- | 2^2.3.273997.5311771.p53 |
| 13 | 61- | 2^2.3.4027.4759.7687.27817.92110001.4672993939.<br>48401662036451.p20 |
| 13 | 63- | 2^2.3^3.43.61.127.337.547.6301.825679.1609669.2714377.<br>5229043.7327657.997294663.p13 |
| 13 | 65- | 2^2.3.53.131.1171.30941.156131.264031.1803647.<br>71442881968439190301.p24 |
| 13 | 67- | 2^2.3.586079017.1093561021297.1195860242597359.p38 |
| 13 | 69- | 2^2.3^2.61.139.1381.10903.282244620282733.<br>2519545342349331183143.p29 |
| 13 | 71- | 2^2.3.6959.12923.201499.p65 |
| 13 | 73- | 2^2.3.14500958610249082921855254822333 6637.p46 |
| 13 | 75- | 2^2.3^2.61.701.1951.4651.9851.30941.161971.<br>2752135920929651.p42 |
| 13 | 77- | 2^2.3.23.419.859.18041.5229043.6249586065506548 22293.p47 |
| 13 | 79- | 2^2.3.3793.16433.6709792556882923.p64 |
| 13 | 81- | 2^2.3^5.61.650971.1609669.57583418699431.<br>1102123844336048491.p42 |
| 13 | 83- | 2^2.3.12451.1383113.p82 |
| 13 | 85- | 2^2.3.103.443.30941.28152511.15798461357509. |

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          11435433293542010176161611.p39
13  87-   2^2.3^2.61.1973.2843.3539.8527.846041103974872866961.
          808648601294417626878199.p35
13  89-   2^2.3.179.9257.716164560927079240026189379811.p62
13  91-   2^2.3.53.10193.34763.264031.1326781.1803647.5229043.
          72019220497491955875703699.p40
13  93-   2^2.3^2.61.311.1117.1303.6377362657.109711631401199502223.
          8170509011431363408568150369.p34
13  95-   2^2.3.191.27361.30941.4986361.12865927.9468940004449.p67
13  97-   2^2.3.389.971.93964390627.p91
13  99-   2^2.3^3.23.61.199.419.859.3169.18041.1609669.
          17551032119981679046729.2192746830056885246381227.p37
13 101-   2^2.3.1213.p109
13 103-   2^2.3.5563.9577825183.299872566772439874463.
          1493185475735966076741431.p56
13 105-   2^2.3^2.43.61.211.337.547.4651.30941.161971.2714377.
          5229043.3357897971.995277238201.707179356161321.p42
13 107-   2^2.3.2141.3286126349184953.p100
13 109-   2^2.3.1091.2617.794018567.7730096009.p96
13 111-   2^2.3^2.61.1481.67495678093.4287755796749.31964044249933.
          5897148699869174859194378551.p53
13 113-   2^2.3.1583.5651.114025363.p110
13 115-   2^2.3.1381.30941.3344431.1309485451.2519545342349331183143.p83
13 117-   2^2.3^3.53.61.79.1093.4603.21841.259507.264031.1519831.
          1609669.1803647.19440694027.57745124662681.
          462116939124761758477693.p35
13 119-   2^2.3.103.443.5229043.13883731.15798461357509.
          1251321793446942324872434004484031.p67
13 121-   2^2.3.23.419.859.18041.4911149.4161245903.1368405847627633.p92
13 123-   2^2.3^2.61.1340947.6740847065723.1374332707191583.
          763604898951326591407.34241383511160276465403.
          2500231913819762291766481.p32
13 125-   2^2.3.251.701.9851.30941.350101751.801560380001.
          2752135920929651.p89
13 127-   2^2.3.5912867.190421635185941.115806724907473978817.p100
13 129-   2^2.3^2.61.6451.119627.7180925126477041.43466212635494347.
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13 131-   2^2.3.263.135979.400852141.p129
13 133-   2^2.3.2129.5229043.12865927.9468940004449.p117
13 135-   2^2.3^4.61.181.2161.4651.30941.161971.203311.650971.
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13 137-   2^2.3.p152
13 139-   2^2.3.62895574665131573770495625630815229.p119
13 141-   2^2.3^2.61.283.1693.183959.1924651.19216136497.
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13 143-   2^2.3.23.53.419.859.18041.264031.1803647.133419712999.
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13 149- 2^2.3.2242046914771.p153
13 151- 2^2.3.2118633909226902785023835895222977.p134
13 153- 2^2.3^3.61.103.443.919.554167.763879.1609669.326881441.
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13 155- 2^2.3.311.1117.30941.100860853831.46458503064581.
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13 157- 2^2.3.80071.27323390907418173038175977.p144
13 159- 2^2.3^2.61.107.19081.194723.189541180943969.
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13 161- 2^2.3.1381.3221.5229043.2519545342349331183143.c144
13 163- 2^2.3.8693117.1279033291.57483449583854824247.p145
13 165- 2^2.3^2.23.61.419.859.2861.4651.18041.30941.161971.
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13 173- 2^2.3.347.113489.481633.p179
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13 189- 2^2.3^4.43.61.127.337.547.757.6301.650971.825679.1609669.
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13 211-  2^2.3.318872063.4532161973764141289403990152729 03.c193
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13 223-  2^2.3.6691.883973.609240789000934129655613721.c211
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Table 13+

| | | |
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| 13 | 4+ | 2.p5 |
| 13 | 5+ | 2.7.11.2411 |
| 13 | 6+ | 2.5.17.p5 |
| 13 | 7+ | 2.7^2.29.p5 |
| 13 | 8+ | 2.p9 |
| 13 | 9+ | 2.7.19.157.271.937 |
| 13 | 10+ | 2.5^2.17.421.601.641 |
| 13 | 11+ | 2.7.p12 |
| 13 | 12+ | 2.14281.p9 |
| 13 | 13+ | 2.7.13417.20333.p5 |
| 13 | 14+ | 2.5.17.p14 |
| 13 | 15+ | 2.7.11.31.157.2411.p8 |
| 13 | 16+ | 2.2657.441281.p9 |
| 13 | 17+ | 2.7.p18 |
| 13 | 18+ | 2.5.17.37.28393.428041.p7 |
| 13 | 19+ | 2.7.p21 |
| 13 | 20+ | 2.41.14281.29881.p12 |
| 13 | 21+ | 2.7^2.29.157.463.22079.p11 |
| 13 | 22+ | 2.5.17.5281.p19 |
| 13 | 23+ | 2.7.47.277.1151.2347.p14 |
| 13 | 24+ | 2.1009.407865361.p15 |
| 13 | 25+ | 2.7.11.101.2411.57751.p16 |
| 13 | 26+ | 2.5.17.380329.p22 |
| 13 | 27+ | 2.7.19.157.163.271.937.904663.p12 |
| 13 | 28+ | 2.113.14281.p25 |
| 13 | 29+ | 2.7.59.1741.8546789918171.p14 |
| 13 | 30+ | 2.5^2.17.421.601.641.28393.460655521.p10 |
| 13 | 31+ | 2.7.373.2729.145831193.p20 |
| 13 | 32+ | 2.193.1601.10433.p26 |
| 13 | 33+ | 2.7.67.157.331.1123.6997.122167.960961.p12 |
| 13 | 34+ | 2.5.17^2.1021.897329.61165661.75094577.p10 |
| 13 | 35+ | 2.7^2.11.29.71.2411.22079.654221.759641.p14 |
| 13 | 36+ | 2.73.4177.14281.181297.815702161.p16 |
| 13 | 37+ | 2.7.223.21017.152219.1548921490187.p17 |
| 13 | 38+ | 2.5.17.229.94621.p33 |
| 13 | 39+ | 2.7.157.13417.20333.79301.p27 |
| 13 | 40+ | 2.407865361.p36 |
| 13 | 41+ | 2.7.83.638453.140299545168523469.p20 |
| 13 | 42+ | 2.5.17.673.2857.4621.28393.23161037562937.p17 |
| 13 | 43+ | 2.7.173.12566837.8001003293.346982008721.p16 |
| 13 | 44+ | 2.89.6073.14281.p39 |
| 13 | 45+ | 2.7.11.19.31.157.271.937.2251.2411.12601.28325071.p20 |
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| 13 | 47+ | 2.7.498851139881.p40 |
| 13 | 48+ | 2.97.2657.88993.441281.283763713.127028743393.p18 |
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13 145+  2.7.11.59.1741.2411.7634343961.8546789918171.
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13 146+  2.5.17.293.466462905277.
         301557549694923141366115643799231359796226965278135642569.p90
13 147+  2.7^3.29.157.463.22079.435709.22896329.54461639.1314082687.
         17712284017.54165939703.14790290915806570339477.
         16049801116294358590175881.p53
13 148+  2.3257.14281.12716161.560678027993.p139
13 149+  2.7.2683.2178083.p156
13 150+  2.5^3.17.421.601.641.28393.460655521.1253653901.1453046401.
         20858983540201.26771688828701.2152338940237584453701.p76
13 151+  2.7.49529.3457568707.2462954753131.c141
13 152+  2.407865361.p161
13 153+  2.7.19.157.271.307.937.1872876367.8122054267.936052510699.
         15840010226107.617886851384381281.
         476622264829847630603684799705499201.p61
13 154+  2.5.17.5281.7393.1702933.150324329.23161037562937.
         718377597171850001.3577574298489429481.
         4209006442599882158485591696242263069.p61
13 155+  2.7.11.373.2411.2729.24181.11268811.145831193.42277560371.
         151091386477111.13496217990809681.16389023943543602257.
         44630257194378716844161.p59
13 156+  2.14281.86113.815702161.2176307537.224277684782113.
         7009558351001043033313937.
         14269421767320773422797054675027409.p73
13 157+  2.7.1571.9421.1579421.1302884463846205672630741.
         5321704544480702707255714477.42086256219133569812191465703.p80
13 158+  2.5.17.317.p172
13 159+  2.7.157.3499.76003.7504072417.202326783229.1462139329561.
         37379721025854083.39349502420618381.
         197795581824045269616226824820047463.p64
13 160+  2.193.1601.10433.96001.68675120456139881482562689.c138
13 161+  2.7^2.29.47.277.1151.2347.22079.84801400975699.
         105533603618353.c134
13 162+  2.5.17.37.109.28393.428041.1471069.16764949.303474601.
         29320472989.4220430741361.1639740678532467913.p102
13 163+  2.7.653.7499.231461.15018169.2756901479.
         2551939308888735197.3645787857397596049.
         346453308677790002393.p95
13 164+  2.14281.c179
13 165+  2.7.11^2.31.67.157.331.1123.2411.6997.53681.58411.82171.
         122167.960961.28325071.128011456717.396089529902881.
         4122652482568228291.159781905331924011841.p65
13 166+  2.5.17.1993.417489312958537.712503393203262887688109.c141
13 167+  2.7.c185
13 168+  2.1009.407865361.16425059281.17254637799169.
         659481276875569.17075564227260177641438981577201118063969.p97
13 169+  2.7.4057.13417.20333.79301.p171
13 170+  2.5^2.17^2.421.601.641.1021.48281.897329.3553001.61165661.
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75094577.4246832221.6152936617.4618988261621.
130262335782633067441.p89
13 171+ 2.7.19^2.157.271.937.845083.3300771062593.2657518772948983.
110463092370899893.170627672083045501.606138721754693161.
104422877883960436477.p73
13 172+ 2.14281.17923777.866662249.149170984032147278296821073.c145
13 173+ 2.7.100512721293404023.33244834894845209424542011.c150
13 174+ 2.5.17.233.349.2437.28393.1321357.14114031000998557.
20970714732554798304809.18178782258903260081683549.
4560345610457374306086188377621.
48898592536658682820603047728758723297.p41
13 175+ 2.7^2.11.29.71.101.2411.16451.22079.57751.654221.759641.
16566575194331.3258254426373251.
132535660270668350675597927401.
302595407569313485796560805198699713897070701.p56
13 176+ 2.353.2657.441281.15020897.21068609.283763713.
19395547354657.29919435299224417.161812513752466240577.c112
13 177+ 2.7.157.4957.5783.8378827.
6521936381117722253551175198473042077230030832125770878689819.
p119
13 178+ 2.5.17.6053.929585777.c184
13 179+ 2.7.359.1433.39062813.p185
13 180+ 2.41.73.4177.4441.8161.14281.29881.181297.217561.815702161.
543124566401.9818892432332713.56156144390197362704881.p107
13 181+ 2.7.47324623.c193
13 182+ 2.5.17.337793.380329.23161037562937.1418792215861230619657.c155
13 183+ 2.7.157.6024266671.298681203493.1535617756259.
5880679831907887.51880092713072077.
2306860055683352406587860779838313.p102
13 184+ 2.407865361.7911246015404132480993.c175
13 185+ 2.7.11.223.2411.21017.152219.888577614401.1548921490187.
1614630622391.10626791079749447.c137
13 186+ 2.5.17.1861.28393.1178621.48534593.111109852618983753193.
576918426137514613314253213249.p134
13 187+ 2.7.10847.421211143.21903343661.128011456717.
367934980978027.617886851384381281.p141
13 188+ 2.14281.41737.553784729353.188172028979257.
398225319299696783138113.7663511503164270157006126605793.c120
13 189+ 2.7^2.19.29.157.163.271.379.463.937.22079.904663.11032183.
704972647.54165939703.762615992953.5588363607297409.
1626764139744163077191689.4948493178819470211350281226827.p72
13 190+ 2.5^2.17.229.421.601.641.94621.
22000710008560364143650941501.
580196961910046805312944783240761.p133
13 191+ 2.7.383.c210
13 192+ 2.257.3230593.36713826768408543617.
3215877717636198473712500018174097551256193.p143
13 193+ 2.7.146540934096087353.c197
13 194+ 2.5.17.289837.1180346441.2249753895467636981.p181
13 195+ 2.7.11.31.157.2411.13417.16381.20333.70981.79301.28325071.
3808876542352598861.584288727345658049575114801.

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        5085423411603888410443263272201.p103
13 196+ 2.113.14281.115249.11096942977.375644987843497.
        42780848865255089.39174253999996064489.
        4803378460849459680406337.c122
13 197+ 2.7.21277.611873120423.c203
13 198+ 2.5.17.37.397.5281.28393.341749.428041.1471069.37869877.
        23057835113017.3577574298489429481.
        416086662911383416679189.p126
13 199+ 2.7.2634761.36130441.602884829.250480083153607.c184
13 200+ 2.401.1201.407865361.45604314401.10381913540858401.
        689249499714233698770401.
        442779263234039928595359287744639041.p123
13 201+ 2.7.157.269.4021.138959.556942300417.3215553577393.
        28376556792667.
        7213463499437577647267326183042302804613669934521.p123
13 202+ 2.5.17.809.20201.4778818919489153480993.
        20307225713395144899769.p172
13 203+ 2.7^2.29^2.59.1741.22079.11367371513.29216756731.
        8546789918171.16397414286709.14913858445348196461.c147
13 204+ 2.137.409.14281.63104137.815702161.4681059934921.
        55444393239496164406865681531894115168657269299195964355161.
        p130
13 205+ 2.7.11.83.2411.638453.1114694881.8325373081.33639770611.
        19643811777631.140299545168523469.45111380897407574171.p136
13 206+ 2.5.17.1237.2473.169156125029.c210
13 207+ 2.7.19.47.157.271.277.829.937.1151.2347.7039.20287.
        84801400975699.
        1577551654677578101258914545922231304801732231.p140
13 208+ 2.2657.145601.441281.283763713.p209
13 209+ 2.7.128011456717.104422877883960436477.p201
13 210+ 2.5^2.17.421.601.641.673.2857.4621.28393.460655521.
        1453046401.23161037562937.61452748127532301.
        296376064934132422053161022580730249078367228427198561.p107
13 211+ 2.7.330963403934881.179988350604280470445878376547.c191
13 212+ 2.14281.92009.18464777.84863647489.
        296102253960054265850194729.4000741506474775723055096281.c155
13 213+ 2.7.157.149191250053.36136869058233840897019.
        3635675840331161742297981783031.
        96292821127287236064998822696599.
        532615479720542238328159944384931.
        1649818592952900908784269998191033146612006407.p60
13 214+ 2.5.17.857.21401.18470838679921.2458739964418553.c201
13 215+ 2.7.11.173.431.2411.14621.12566837.1888302431.8001003293.
        346982008721.9391016904700369.370696297879940727105731.p148
13 216+ 2.1009.3889.53569.680401.52883713.407865361.29975087953.
        659481276875569.923563008624961.6654909974864689.
        558181416418089697.p133
13 217+ 2.7^2.29.373.2729.11719.22079.145831193.
        16389023943543602257.27340842173064358257699651251.
        9265063882320201898324947759713.p138
13 218+ 2.5.17.p241
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13 219+  2.7.157.45553.64803179963.110628894631.
         5030641640221783826510884729079173775636718045309795338674775249_
         9.
         p150
13 220+  2.41.89.881.6073.14281.29881.39161.4476745241.543124566401.
         668229430151772307653060812800711311233.c162
13 221+  2.7.13417.20333.23869.79301.741677.1566449.194388949.
         617886851384381281.1590964049817874064437613134993537.c156
13 222+  2.5.17.149.6217.14653.28393.201091153.1738568407946597.
         596131104371449237.72899319864895280400157.
         120687541344843078804469.
         1438734846120969865240176038964493.
         186121273917021854408917552512305587532503574509.p63
13 223+  2.7.13381.13163260466767.c231
13 224+  2.193.449.1601.10433.83777.114689.58317286721.
         10199228225275634431937.10759970447698109015939009.
         68675120456139881482562689.p144
13 225+  2.7.11.19.31.101.151.157.271.937.2251.2411.11551.12601.
         57751.2113801.28325071.966623849742301.3258254426373251.
         101348453341211701.19145097390154976011.c134
13 226+  2.5.17.3617.16273.23957.2245321301.c229
13 227+  2.7.41050444705991995903280091731.c224
13 228+  2.457.761.2281.14281.20521.692513.815702161.
         62300665486585624081.
         2135382121254983685021572341095722302254446826817.c154
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For pointers to the Tables $14-, 14+, \ldots, 98-, 98+$, see §3.

Table 99-

| | | |
|---|---|---|
| 99 | 1- | 2.7^2 |
| 99 | 3- | 2.7^2.9901 |
| 99 | 5- | 2.7^2.p8 |
| 99 | 7- | 2.7^3.12979.p8 |
| 99 | 9- | 2.7^2.19^2.127.9901.p8 |
| 99 | 11- | 2.7^2.397.p18 |
| 99 | 13- | 2.7^2.53.131.157.p18 |
| 99 | 15- | 2.7^2.9901.97039801.p16 |
| 99 | 17- | 2.7^2.320417.4653343.p20 |
| 99 | 19- | 2.7^2.571.39740734591141.p20 |
| 99 | 21- | 2.7^3.43.9901.12979.10468417.p23 |
| 99 | 23- | 2.7^2.30466170347.222628529119.p23 |
| 99 | 25- | 2.7^2.97039801.5595086438943451.p25 |
| 99 | 27- | 2.7^2.19^2.127.811.919.9901.20535283.30622807.p23 |
| 99 | 29- | 2.7^2.523.162517.702787.175281266965559.p28 |
| 99 | 31- | 2.7^2.39371.p56 |
| 99 | 33- | 2.7^2.397.661.9901.1083721.230128580234081233.p32 |
| 99 | 35- | 2.7^3.9241.12979.10468417.97039801.1516445266992375820241.p23 |
| 99 | 37- | 2.7^2.p72 |
| 99 | 39- | 2.7^2.53.79.131.157.9901.63495043.821456624786426851.p39 |
| 99 | 41- | 2.7^2.83^2.76261.3036275993.p62 |
| 99 | 43- | 2.7^2.1549.348207551.p73 |
| 99 | 45- | 2.7^2.19^2.127.9901.20535283.97039801.3551863449061. |
| | | 716359512559681.9134249824299601.p21 |
| 99 | 47- | 2.7^2.p92 |
| 99 | 49- | 2.7^4.12979.843781.6088153.10468417.10867907.p64 |
| 99 | 51- | 2.7^2.9901.320417.4653343.57688814829868263071.p64 |
| 99 | 53- | 2.7^2.107.118566886147247.p88 |
| 99 | 55- | 2.7^2.397.42901.204601.97039801.230128580234081233. |
| | | 327414198153783782669101.p47 |
| 99 | 57- | 2.7^2.571.9901.1676029.39740734591141.37151009801325375691.p66 |
| 99 | 59- | 2.7^2.1063.148537384959989.4631579044347830647.p80 |
| 99 | 61- | 2.7^2.8663.27817.105653.238877.49036900943. |
| | | 10527303441917105235878602048817.p60 |
| 99 | 63- | 2.7^3.19^2.43.127.4789.9901.10333.12979.2729413.10468417. |
| | | 20535283.193620897667.296323515978335648713. |
| | | 20405404499169396293707.p26 |
| 99 | 65- | 2.7^2.53.131.157.97039801.1734834401.821456624786426851. |
| | | 550780160268332441039460497501.p57 |
| 99 | 67- | 2.7^2.269.51134669.416950783.12756973500253. |
| | | 3780574316663829815529867056872778269.p64 |
| 99 | 69- | 2.7^2.139.9901.15878419.30466170347.222628529119. |
| | | 536561938621.28820089845571.242460345918040491433. |
| | | 11939460883273302109457.p33 |
| 99 | 71- | 2.7^2.14627.158047.15490781.482044561.17179549507. |
| | | 6787613924430123425819.2123069070360192515833718179.p56 |
| 99 | 73- | 2.7^2.36793.3278172880914712338521191295531.p109 |
| 99 | 75- | 2.7^2.151.9901.34784401.97039801.5595086438943451. |
| | | 9134249824299601.1461830744902104296233051.p71 |

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99  77-  2.7^3.397.12979.5526137.10468417.113089684775453.
         32982101201754013.230128580234081233.
         86985201890916304423611454398165553.p49
99  79-  2.7^2.3319.6637.428023.853429417.
         27451277850846421626828043747 7.
         27100756722615202899087603064941702173.p67
99  81-  2.7^2.19^2.127.811.919.1621.9901.20535283.30622807.
         36563868632477911923127.p105
99  83-  2.7^2.167.12119.106777977100687331.p141
99  85-  2.7^2.2551.3061.320417.4653343.97039801.410043401.
         57688814829868263071.p113
99  87-  2.7^2.523.9901.57073.69427.162517.702787.175281266965559.
         7281734814760575782986405667.p103
99  89-  2.7^2.1462102202783.983575267881704300649386383.c137
99  91-  2.7^3.53.131.157.1093.12979.721267.10468417.41256851819.
         821456624786426851.p125
99  93-  2.7^2.9901.39371.876619.902007055801.229328611909843.
         1380833403520648186 81.
         18979663619975067593590717641897091789568243789952868031.p68
99  95-  2.7^2.571.3041.97039801.2334201491.25872120641.
         39740734591141.3715100980132537 5691.20705547235683560491 1.p101
99  97-  2.7^2.30136543.222376963.89974471973.c165
99  99-  2.7^2.19^2.127.397.661.9901.251857.1083721.20535283.
         47803141.230128580234081233.
         1130254570964927104835675831 3821.p107
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Table 99+

| | | |
|---|---|---|
| 99 | 1+ | 2^2.5^2 |
| 99 | 2+ | 2.13^2.29 |
| 99 | 3+ | 2^2.5^2.31.313 |
| 99 | 4+ | 2.2617.p5 |
| 99 | 5+ | 2^2.5^3.p8 |
| 99 | 6+ | 2.13^2.29.p8 |
| 99 | 7+ | 2^2.5^2.p12 |
| 99 | 8+ | 2.17.1553.250993.p6 |
| 99 | 9+ | 2^2.5^2.31.37.313.39097.p6 |
| 99 | 10+ | 2.13^2.29.61.821.p12 |
| 99 | 11+ | 2^2.5^2.23.52757.250031.p9 |
| 99 | 12+ | 2.2617.18353.p16 |
| 99 | 13+ | 2^2.5^2.521.p22 |
| 99 | 14+ | 2.13^2.29.113.1429.11472833.p12 |
| 99 | 15+ | 2^2.5^3.31.181.313.6271.33751.243301.p8 |
| 99 | 16+ | 2.449.p29 |
| 99 | 17+ | 2^2.5^2.137.p30 |
| 99 | 18+ | 2.13^2.29.30637.96049801.2198833093.p11 |
| 99 | 19+ | 2^2.5^2.647.p34 |
| 99 | 20+ | 2.41.601.2617.18353.688201.99013241.p14 |
| 99 | 21+ | 2^2.5^2.31.313.189253.932065347907.p19 |
| 99 | 22+ | 2.13^2.29.89.p38 |
| 99 | 23+ | 2^2.5^2.47.5122331.2294665013.p27 |
| 99 | 24+ | 2.17.97.1553.6337.250993.696257.418258071409.p15 |
| 99 | 25+ | 2^2.5^4.3301.947651.19019801.p30 |
| 99 | 26+ | 2.13^3.29.65071241.p39 |
| 99 | 27+ | 2^2.5^2.31.37.163.313.2377.39097.650827.p31 |
| 99 | 28+ | 2.281.2617.18353.12625714428146384294689.p24 |
| 99 | 29+ | 2^2.5^2.59.929.180174217.p43 |
| 99 | 30+ | 2.13^2.29.61.821.6121.1118041.96049801.184231655921.p23 |
| 99 | 31+ | 2^2.5^2.387253.1330147009.140028543787291.941091948776813.p17 |
| 99 | 32+ | 2.193.3137.105481090085456435565163841.p32 |
| 99 | 33+ | 2^2.5^2.23.31.67.313.22639.52757.250031.295110971.<br>5615902187993659.p18 |
| 99 | 34+ | 2.13^2.29.549320729.p56 |
| 99 | 35+ | 2^2.5^3.71.2311.19019801.932065347907.23622410172131.p30 |
| 99 | 36+ | 2.73.433.1873.2017.2617.8209.18353.24697.837802513.<br>9227446848219601.p20 |
| 99 | 37+ | 2^2.5^2.p72 |
| 99 | 38+ | 2.13^2.29.9324916217.9879998861.350580803333.<br>144453687390242609.p24 |
| 99 | 39+ | 2^2.5^2.31.313.521.82911940969819963.<br>1684301387713950072653.p31 |
| 99 | 40+ | 2.17.1553.250993.624241.696257.225617921.p50 |
| 99 | 41+ | 2^2.5^2.p80 |
| 99 | 42+ | 2.13^2.29.113.1429.21169.11472833.96049801.478406070661.p44 |
| 99 | 43+ | 2^2.5^2.1646946417182137.p69 |
| 99 | 44+ | 2.2617.18353.3796673.34280401.180921137.p58 |
| 99 | 45+ | 2^2.5^3.31.37.181.313.2161.6271.33751.39097.46171.243301. |

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          650827.19019801.28556470441.p30
99  46+   2.13^2.29.271861.12993609334073.47686397099278076441.p50
99  47+   2^2.5^2.13260878740517132985360700296254354341931.p52
99  48+   2.449.577.1366841761.1096939914835164l537.
          94817123729941607978286682849.p33
99  49+   2^2.5^2.932065347907.28126718074643352 79.p66
99  50+   2.13^2.29.61.101.401.701.821.184231655921.
          46259978873311450015201.p50
99  51+   2^2.5^2.31.103.137.313.38573137.10138830384564590312437.
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99  52+   2.2617.17681.18353.493505039561.65797600936601.
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99  53+   2^2.5^2.4571887.28554336292649444913139664 2507.
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99  54+   2.13^2.29.109.4861.30637.96049801.2198833093.13157816761.
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99  55+   2^2.5^3.23.52757.250031.19019801.295110971.656450787631.
          13474326747311.437636575693447830130625591.p29
99  56+   2.17.1553.250993.696257.563330705804744753.
          2313004363558593378056 33.p55
99  57+   2^2.5^2.31.229.313.647.775530570242561500537.
          524295109764750246494 47.7554768351678397002375417 1.p34
99  58+   2.13^2.29^2.25870321.90829662977.651293410628341 3.
          272497433337357937.989142227744415598 9.p40
99  59+   2^2.5^2.709.937060303.49112638873.21271793245780759751.p74
99  60+   2.41.601.1801.2617.18353.688201.99013241.69519309001.
          50710302877921.234346431711121.309640814187841.
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99  61+   2^2.5^2.53681.315150541115619161845399 81.
          1912765952076108379367208007843279516683557 33.p46
99  62+   2.13^2.29.1117.7069.
          1317003917633241579350563350934673393290189955 7017.p64
99  63+   2^2.5^2.31.37.313.39097.189253.650827.99387080101.
          932065347907.27157471213321.47309016581769557 17.
          19387094798099329057.p29
99  64+   2.257.p126
99  65+   2^2.5^3.521.2861.19019801.168430138771395007 2653.
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99  66+   2.13^2.29.89.16568641.59656081.96049801.288211003609.
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99  67+   2^2.5^2.23210569806111026669 3.
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99  68+   2.2617.18353.4535080073.322776284081.15032031798473.
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99  69+   2^2.5^2.31.47.313.466717.5122331.54783793.2294665013.
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99  70+   2.13^2.29.61.113.821.1429.42701.11472833.184231655921.
          478406070661.173435653848783494767 21.p69
99  71+   2^2.5^2.p140
99  72+   2.17.97.1553.6337.250993.696257.2462155057.418258071409.
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99  73+ 2^2.5^2.
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99  74+ 2.13^2.29.149.p142
99  75+ 2^2.5^4.31.181.313.3301.6271.33751.243301.947651.19019801.
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99  76+ 2.2617.18353.26033801.677502728081.883632897055732609l297.
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99  77+ 2^2.5^2.23.463.617.2003.45893.52757.250031.797567.
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        191529723322794734369l533.p53
99  78+ 2.13^3.29.43615261.65071241.96049801.706989037.
        92868390664377312052526777061680072839 7.p80
99  79+ 2^2.5^2.317.19218236092543.15802135105345814 59.
        32167184491150599647908 39.294499246623835031714799759 53.p69
99  80+ 2.449.38561.40801.29236769680959656707974873 61.
        9481712372994160797828668 2849.
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99  81+ 2^2.5^2.31.37.163.313.487.2377.7129.39097.650827.675541.
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99  82+ 2.13^2.29.342761.83608712544984558637.
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99  83+ 2^2.5^2.38300351.2978746923617.
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99  84+ 2.281.2617.18353.127849.2460783697.9227446848219601.
        126257144281463842946 89.221453415989744487722489.p82
99  85+ 2^2.5^3.137.34511.19019801.332067621806931431.
        20842757140684933502826 1.615287002470019622809170998441.p83
99  86+ 2.13^2.29.173.720550927725 7.p153
99  87+ 2^2.5^2.31.59.313.929.180174217.
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        7565894381225637658169837392669593442248967.p74
99  88+ 2.17.353.1553.250993.696257.c158
99  89+ 2^2.5^2.179.2671.16264346875446607587832 09.
        1006044261856165162030102 13.p120
99  90+ 2.13^2.29.61.821.6121.30637.1118041.96049801.2198833093.
        13157816761.184231655921.1244306337458411850384 1.
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99  91+ 2^2.5^2.521.6007.932065347907.16843013877139500726 53.
        569758511913346002847 7.6961703058707154579277.p97
99  92+ 2.2617.5153.18353.7788353.38116889.59142569.95281273.
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99  93+ 2^2.5^2.31^2.313.387253.267099163.1330147009.
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99  94+ 2.13^2.29.2633.82721.567949.1512775203058949909.
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99  95+ 2^2.5^3.191.647.5264521.19019801.
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99  96+ 2.193.3137.1054810900854564355651638 41.
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99 100+ 2.41.601.2617.18353.688201.99013241.50710302877921.
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